

# Finite size effects in pair production processes

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1 Introduction

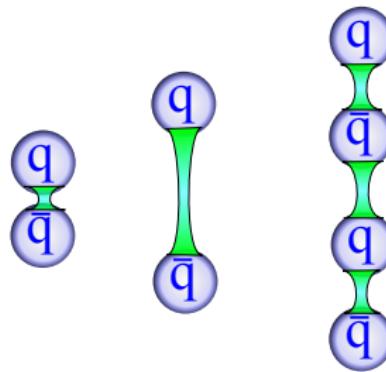
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3 Applications

# QCD Motivation

From QCD:

The success of color rope/string models in describing Heavy Ion collisions (e.g. at LHC):



Quark potential is linear with separation: if a  $q - \bar{q}$  pair is separating, the interaction creates more and more quark pairs until energy is depleted.



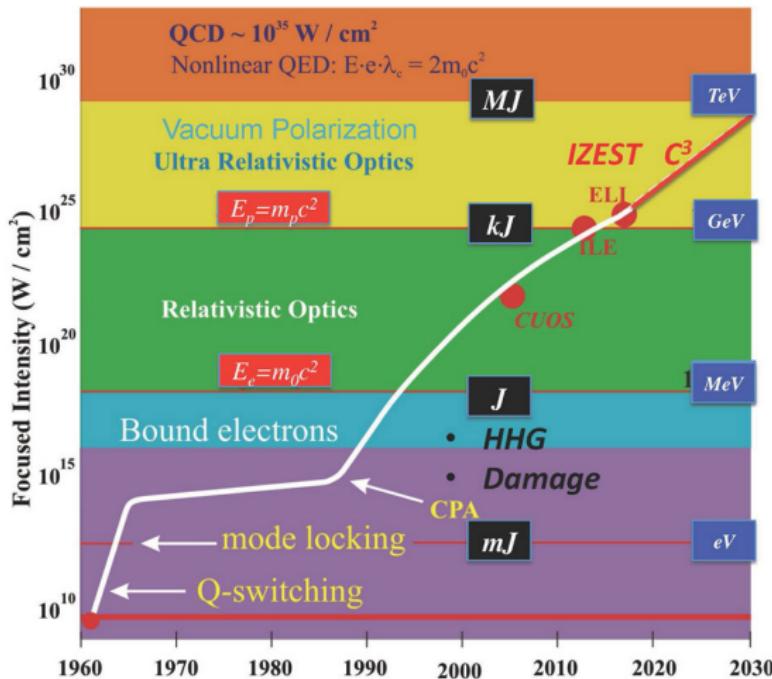
# QED Motivation

- QED pair production from vacuum was predicted long ago, but was not yet observed → "Holy Grail of QED".
- QED pair production may take place in near miss heavy ion collisions.

QED pair production near massive astrophysical objects (black holes, magnetars, possible source of gamma ray bursts?)



# Laser Technology



For a list of present and upcoming facilities see: A. Di Piazza et. al, Rev. Mod. Phys. vol. 84, (2012) 1177



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# Theory

Relevant scales:

- Field strength:

$$\mathcal{E}_c = \frac{m^2 c^3}{e\hbar}$$

- Time:

$$t_c = \frac{\hbar}{mc^2}$$

- Frequency:

$$\omega_c = \frac{e\mathcal{E}}{mc}$$

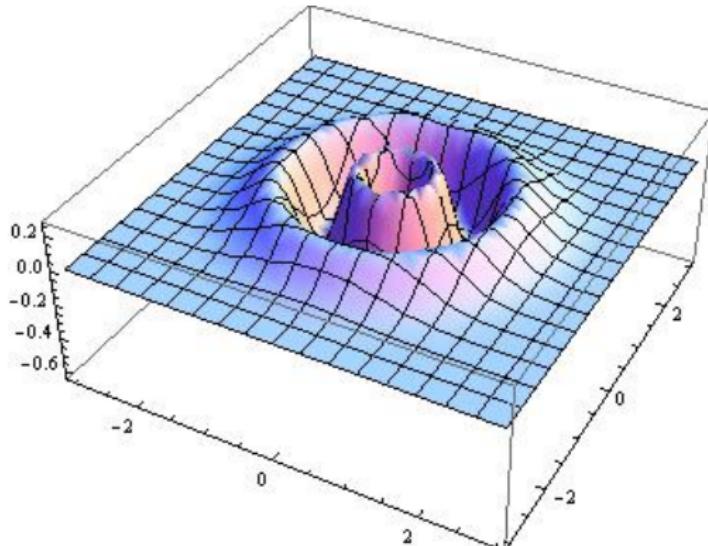
- Spatial gradient:

$$\partial_r = \frac{mc}{\hbar}$$

# Wigner function

Tool of description: the Wigner function:

- Quantum analogue of the classical phase space distribution.



Wigner function of an  $n=3$  Fock state.

# Wigner function

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{\rho}(\vec{x}, \vec{s}, t) = e^{-ig \int_{-1/2}^{1/2} \vec{A}(\vec{x} + \lambda \vec{s}, t) \cdot \vec{s} d\lambda} \left[ \Psi(\vec{x} + \frac{\vec{s}}{2}, t), \bar{\Psi}(\vec{x} - \frac{\vec{s}}{2}, t) \right] \quad (1)$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x}, \vec{p}, t) = -\frac{1}{2} \int e^{-i\vec{p}\vec{s}} \langle \Omega | \hat{\rho}(\vec{x}, \vec{s}, t) | \Omega \rangle d^3 s \quad (2)$$



# Wigner function

The evolution equation:

$$D_t W = -\frac{1}{2} \vec{D}_{\vec{x}} [\gamma^0 \vec{\gamma}, W] - i m [\gamma^0, W] - i \vec{P} \{ \gamma^0 \vec{\gamma}, W \} \quad (3)$$

The equation has the following non-local differential operators:

$$D_t = \partial_t + g \vec{\mathcal{E}}(\vec{x}, t) \vec{\nabla}_{\vec{p}} - \frac{g \hbar^2}{12} (\vec{\nabla}_{\vec{x}} \vec{\nabla}_{\vec{p}})^2 \vec{\mathcal{E}}(\vec{x}, t) \vec{\nabla}_{\vec{p}} + \dots \quad (4)$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + g \vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} - \frac{g \hbar^2}{12} (\vec{\nabla}_{\vec{x}} \vec{\nabla}_{\vec{p}})^2 \vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + \dots \quad (5)$$

$$\vec{P} = \vec{p} + \frac{g \hbar}{12} (\vec{\nabla}_{\vec{x}} \vec{\nabla}_{\vec{p}}) \vec{\mathcal{B}}(\vec{x}, t) \times \vec{\nabla}_{\vec{p}} + \dots \quad (6)$$

For spin-1/2, the 4x4 gamma matrix basis is used:

$$W(x, p, t) = \frac{1}{4} [\mathbb{1} \mathbb{s} + i \gamma_5 \mathbb{p} + \gamma^\mu \mathbb{v}_\mu + \gamma^\mu \gamma_5 \mathbb{a}_\mu + \sigma^{\mu\nu} \mathbb{t}_{\mu\nu}] \quad (7)$$

# Equations of motion for the spin-1/2 Wigner function

We arrive at a system for 16 unknown real functions:

$$D_t \mathbb{S} - 2\vec{P} \cdot \vec{t}_1 = 0 \quad (8)$$

$$D_t \mathbb{P} + 2\vec{P} \cdot \vec{t}_2 = 2m \mathbf{a}_0 \quad (9)$$

$$D_t \mathbb{V}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbf{v}} = 0 \quad (10)$$

$$D_t \mathbf{a}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbf{a}} = 2m \mathbb{P} \quad (11)$$

$$D_t \vec{\mathbf{v}} + \vec{D}_{\vec{x}} \mathbb{V}_0 + 2\vec{P} \times \vec{\mathbf{a}} = -2m \vec{t}_1 \quad (12)$$

$$D_t \vec{\mathbf{a}} + \vec{D}_{\vec{x}} \mathbf{a}_0 + 2\vec{P} \times \vec{\mathbf{v}} = 0 \quad (13)$$

$$D_t \vec{t}_1 + \vec{D}_{\vec{x}} \times \vec{t}_2 + 2\vec{P} \mathbb{S} = 2m \mathbb{V} \quad (14)$$

$$D_t \vec{t}_2 - \vec{D}_{\vec{x}} \times \vec{t}_1 - 2\vec{P} \mathbb{P} = 0 \quad (15)$$



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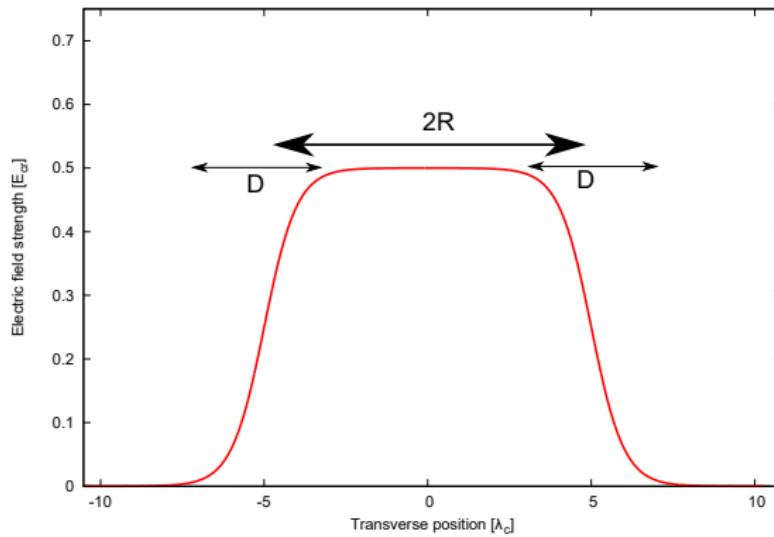
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# Inhomogeneous model field

We would like to investigate the combined effect of space/time dependence.

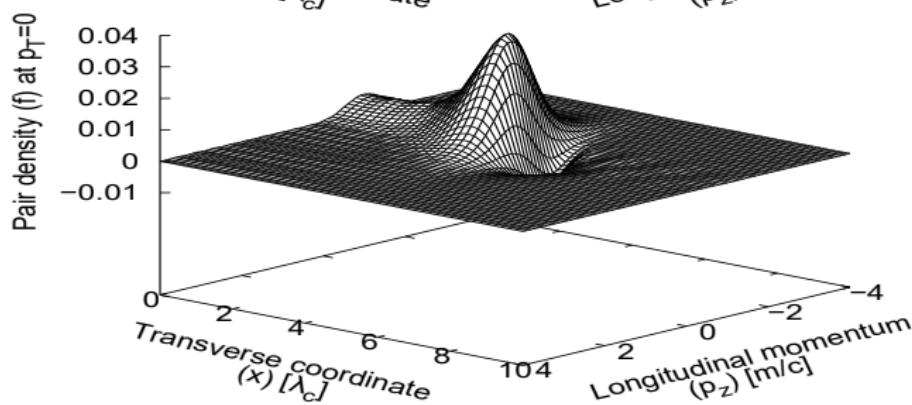
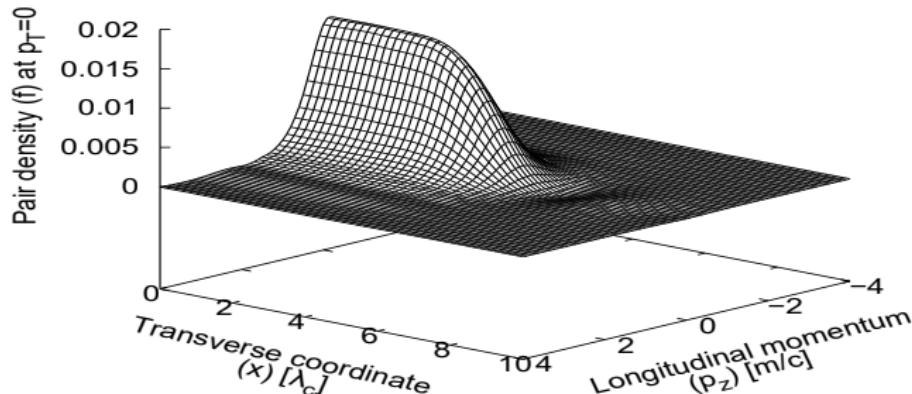
Consider an inhomogeneous plateau field in the transverse spatial direction and Sauter-like ( $\cosh(t/\tau)^{-2}$ ) time dependence. ( $E_0 = 0.5E_{cr}$ ).



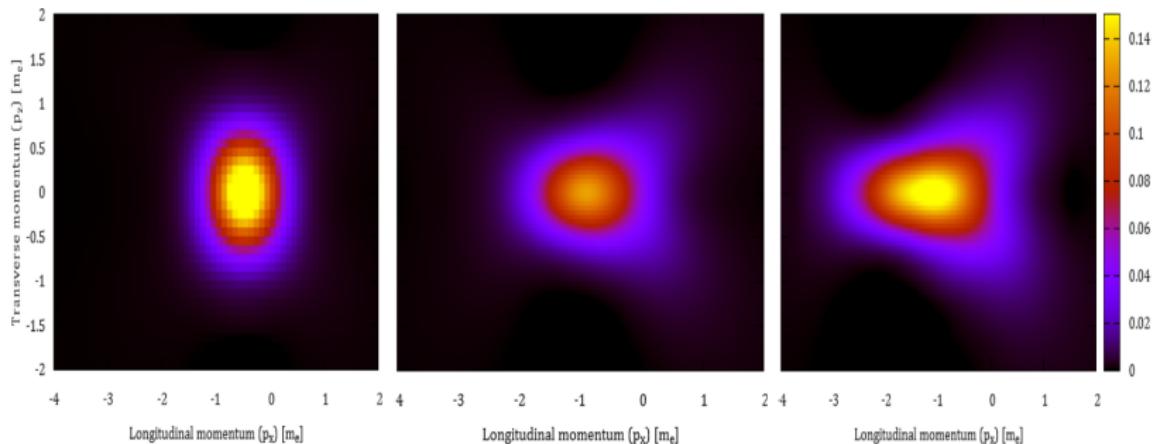
D. Berényi et al, Physics Letters B, Vol 749, pp. 210-214, DOI: 10.1016/j.physletb.2015.07.074.



# Inhomogeneous model field



# Inhomogeneous model field



$$(a) \tau = 1 \frac{\lambda_c}{c}$$

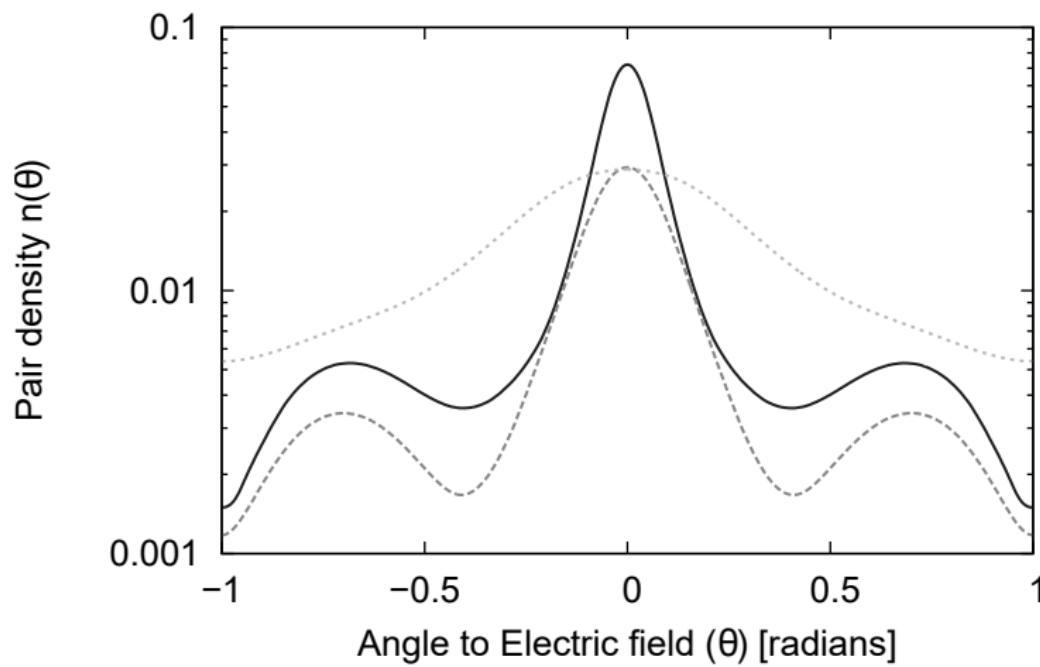
$$(b) \tau = 2 \frac{\lambda_c}{c}$$

$$(c) \tau = 3 \frac{\lambda_c}{c}$$

# Inhomogeneous model field

What would a calorimetric experiment see?

Longitudinal angle to the Electric field ( $\theta$ , with  $\theta = 0$  is longitudinal direction,  $\theta = \pm\frac{\pi}{2}$  is the transverse direction):



# Inhomogeneous model field

Behaviour is similar to the bifurcation predicted in quantum optics for the Wigner function of a squeezed state :  $\Psi(x) \propto e^{-(s/2)(x-\sqrt{2}a)}$

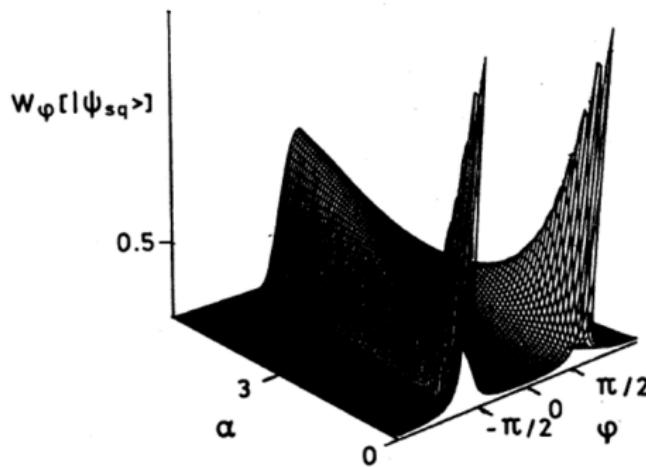
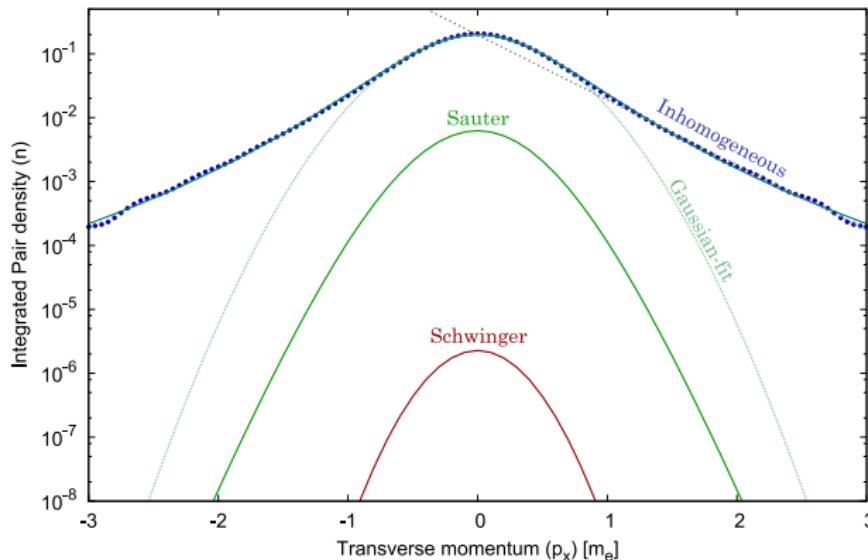


FIG. 1. The probability,  $W_\varphi[|\psi_{sq}\rangle]$ , to find the phase of an oscillator to lie between the value  $\varphi$  and  $\varphi + d\varphi$  when the oscillator is in a state  $|\psi_{sq}\rangle$ , Eq. (3), highly squeezed in the  $x$  variable ( $s = 21$ ) undergoes a bifurcation for decreasing displacement  $\alpha$

# Inhomogeneous model field

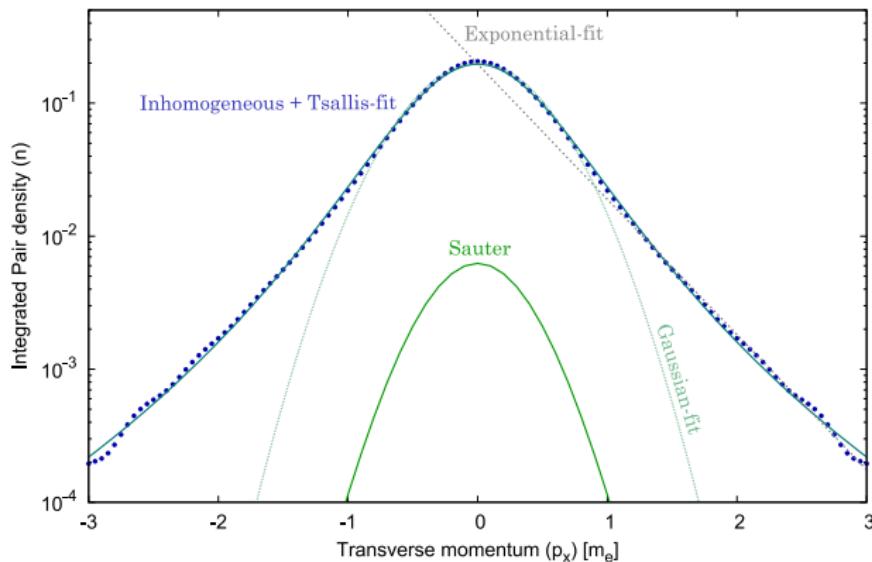
How does the transverse spectra compares to the homogeneous models?



Gaussianity:  $Ae^{-\beta p_x^2}$ , with  $\beta_{\text{Schwinger}} = 6.6$ ,  $\beta_{\text{Sauter}} = 4.3$ ,  $\beta_{\text{InHom}} = 2.6$

# Inhomogeneous model field

How does the transverse spectra compares to the homogeneous models?



$$\text{Tsallis: } A \left( 1 + (1 - q) \frac{p_x}{p_{x0}} \right)^{\frac{1}{1-q}}, \text{ with } q = 1.1, p_{x0} = 0.26$$

# Inhomogeneous model field

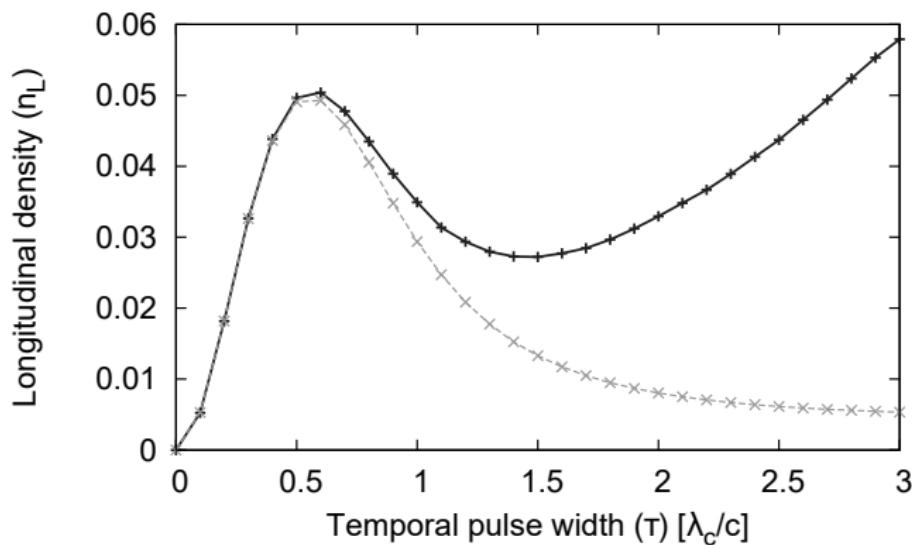
## Observations:

- The inhomogeneity increases the particle yield.
- The increase is further enhanced if the pulse lasts longer (as opposed to the homogeneous case).
- As a consequence, homogeneous models of string fragmentation may underestimate the particle production rates.
- The transverse spectra develops distinct higher-momenta "shoulders" at the boundaries.



# Inhomogeneous model field

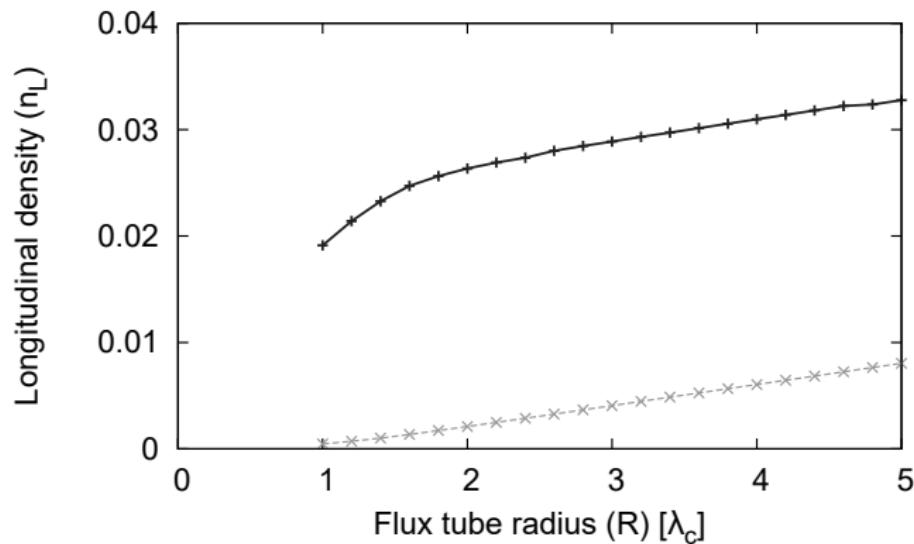
Integrated particle yields:



Again: increase with pulse length!

# Inhomogeneous model field

Integrated particle yields:



Same slope: inhomogeneous pair production is a surface effect!  
Predicted by Heisenberg in 1934!

W. Heisenberg, Sachsische Akademie der Wissenschaften, Vol. 86, p. 317 (1934)



# Summary

- The evolution of the Wigner function in multiple dimensions is possible including space and time dependent field configurations.
- We discussed a simple model of gluon strings that model the interplay of space and time localizations and showed characteristic differences from homogeneous results:
- Differences in transverse and longitudinal spectra
- Potentially measurable momentum angle distribution difference
- Different scaling with temporal duration

D. Berényi et al, Physics Letters B, Vol 749, pp. 210-214, DOI: 10.1016/j.physletb.2015.07.074.

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